Measuring Middle-School Teachers' Mathematical Knowledge for Teaching Rate and Proportionality

Measuring Middle-School Teachers' Mathematical Knowledge for Teaching Rate and Proportionality

Nicole Shechtman, Jennifer Knudsen, Jeremy Roschelle, Geneva Haertel, Lawrence Gallagher, Kenneth Rafanan, and Philip Vahey
SRI International

April 8, 2006

AERA Roundtable Discussion

To contact the authors: nicole.shechtman@sri.com

Suggested Discussion Topics

1. What is the content being assessed, and what type of items were created to address these topics? *For more info, please see:*
    - Intervention (Pages 1-3)
    - Content Domain and Sample Item (Pages 9-11)
    - Preliminary Findings (Pages 6-8)

2. How was the assessment developed and validated? *For more info, please see:*
    - Development and Validation (Pages 5-6)

3. How does this research fit into the overall MKT literature? *For more info, please see:*
    - Appendix (Page 13)

Background

The overarching purpose of the Scaling Up SimCalc research program is to test at scale the following hypothesis:

> *A wide variety of middle school teachers can use an innovative integration of technology and curriculum to create opportunities for their students to learn complex and conceptually difficult mathematics.*

Specifically, we are investigating if teachers from across the state of Texas can use a specially designed replacement unit and SimCalc software to help their students learn mathematics important to the Texas state frameworks, as well as content that goes beyond the state framework—the beginning pieces of the "mathematics of change," leading to Calculus.

The specific components of our intervention provide access for teachers and students to important mathematics through SimCalc. The components include:

- A replacement unit that addresses state standards and specialized "mathematics of change" content in an easy-to-use form for teachers.
- SimCalc's Java Math Worlds software that uses simulations of motion (and general accumulation) and dynamically linked representations.
- A three-part training, including a summer and fall session, totaling six days.
- Availability of software and curriculum trouble-shooting during the school year.
- Teachers' use of the materials for 2-3 weeks in their classroom, replacing the materials they would usually use to teach the same content.

We are testing our hypothesis using a randomized experimental design. Our primary outcome measure is student learning of mathematics. "Math knowledge for teaching" (MKT) is also a potentially important variable. We measure teacher MKT using a paper-and-pencil assessment administered before and after the summer workshop. We will use results from this assessment to determine: (a) what relevant mathematics, if any, teachers learn during the summer workshops, and (b) the extent to which teachers' MKT is related to student outcomes.

The conceptual framework of our assessment was inspired by the work of Ball and Hill, who have done pioneering work conceptualizing the specialized form of mathematics necessary to teach mathematics. The specific content of our assessment is closely related the content of our particular curricular materials. Our assessment was designed to measure:

- The mathematics knowledge relevant to the teachers' teaching of the unit, so that their middle school students learn the target content
- The mathematics teachers could learn through our relatively short training, focused on teachers' use of our curricular materials and software

In what follows, we present and explain the design of our curricular materials, the design of the overall experiment, the role of the MKT assessment in this experiment, and the creation and validation processes of this assessment.

The Intervention

Content Focus

SimCalc is based on the notion that children should have access to the "mathematics of change"—a succession of ideas that culminates in the concepts of Calculus—as early as possible in school. The focus of SimCalc aligned with content at the middle school level is proportionality. Proportionality is a central, crosscutting theme that involves number and operations, geometry, and algebra, and connections among them (Hiebert & Behr, 1988) and is a large focus area in both the National Council of Teachers of Mathematics (NCTM) and state standards (NCTM, 2000). Our emphasis on rate focuses on the mathematics of change.

Materials

SimCalc MathWorlds was designed to help middle and high school students make the connections required to understand concepts central to the mathematics of change. The software makes dynamic links between simulations of moving objects (linear motion), the associated position and velocity graphs, and tables and formulas. Through exploring challenging problems, students make rich connections among these representations, resulting in their understanding of rate of change.

Figure 1 shows two windows in MathWorlds representing the same phenomenon. The simulation window shows two runners moving across a field. The position graph window shows a plot of distance (from start) versus time, with lines representing each runner's movement created in synchronization with the simulation. Students can change the speed of the runners by changing the slope of their associated lines. They can also create more complex motions comprised of linear segments. Additionally, students can run the simulation in discrete steps and drop marks in order to apply numerical reasoning to the situation. While not shown in this figure, tables and formulas are also available to students. Velocity graphs, showing plots of speed versus time, are important for building an intuitive understanding of calculus and are included in other SimCalc interventions. Fuller descriptions of the software and associated materials are available elsewhere (Kaput & Roschelle, 1998; Roschelle & Kaput, 1996; Roschelle, Kaput, & Stroup, 2000).

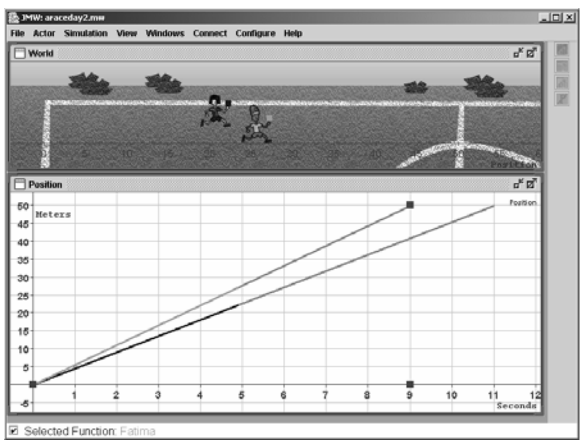


Figure 1. MathWorlds simulation and line graph windows

For the Scaling Up SimCalc intervention, we use a replacement unit strategy, which has been shown to be successful at large scale in mathematics reform in California (e.g., see Cohen & Hill, 2001). Our replacement unit, *Managing the Soccer Team*, takes a linear function (y=kx) approach to rate and proportionality (Greenes & Findell, 1999), exploiting SimCalc curriculum and MathWorlds software. Through examination of a number of situations involving motion or accumulation, students learn how y=kx can be used to express change, how k describes the rate of change, the connections among representations of that change, and how to solve problems integrating all of these.

Experimental Design

Scaling Up SimCalc uses a delayed treatment randomized experimental design (Campbell, Shadish, & Cook, 2001; Slavin, 2002), in which teachers are randomly assigned to one of two conditions: Immediate Treatment (IT) and Delayed Treatment (DT). This design affords the implementation of a control group, as well as equity among research participants. Both the IT and DT groups receive treatment interventions; however, the treatment for the DT group is always delayed by one year. The DT group therefore serves as the IT's control group in any given year. To contrast with the impacts of the treatment intervention, the experimental control is designed to have teachers teach material comparable to what they would teach under typical circumstances.

The basic experimental design is as follows (Figure 2). In summer 2005, both groups attended a 2-day preparatory workshop. This preparatory workshop was an already existing workshop, Texteams, used in Texas to help teachers understand how to teach proportionality using linear functions (y = kx). After this workshop, DT teachers departed, while IT teachers attended a 3-day SimCalc workshop. In the fall, IT teachers come together once again for a weekend planning workshop. During the school year, IT teachers are asked to teach the replacement unit, *Managing the Soccer Team*, in the place of their usual rate and proportionality unit, while DT teachers are asked to teach rate and proportionality as usual.

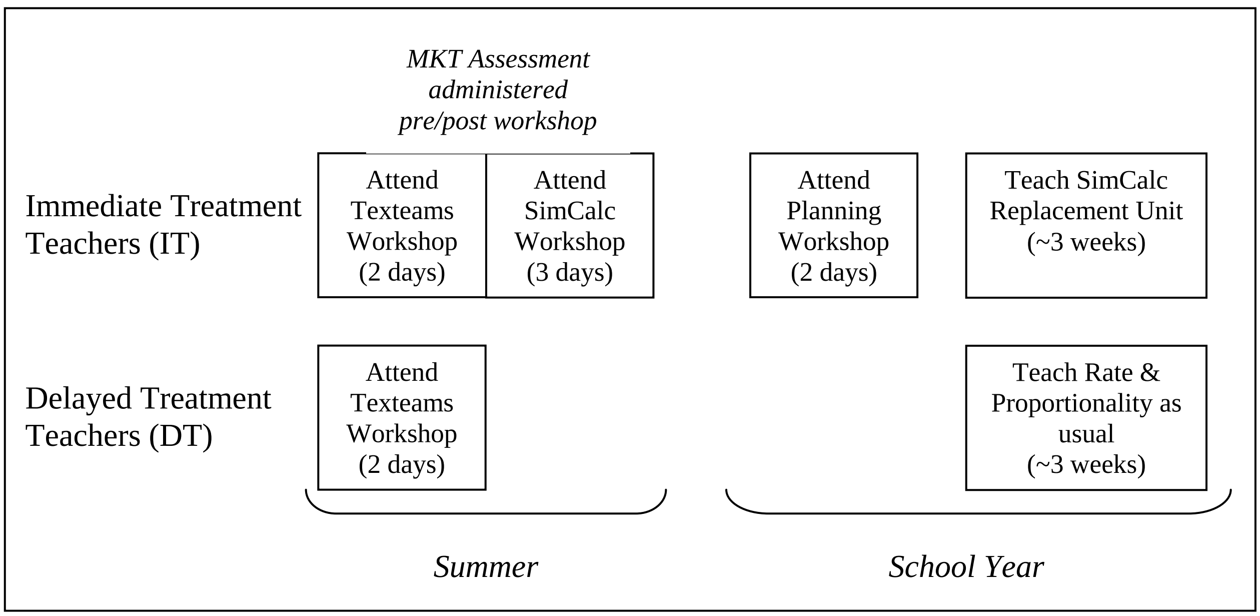


Figure 2. Scaling Up SimCalc first-year research design (began Summer 2005)

In summers 2006 and 2007, IT teachers will be presented with further opportunities for training, and DT teachers always receive identical opportunities with a 1-year lag.

Within this design, we administer the MKT assessment to both groups at two points: pre- and post-summer workshop (after 2 days in the DT group and 5 days in the IT group).

Several other key measures are taken throughout the year. The student assessment is administered in both conditions pre- and post unit. It was designed to measure learning gains over the 3-week units taught in both the IT and DT conditions. Other measures include classroom observations, teacher logs during the unit, teacher surveys about attitudes toward teaching and student capabilities, and in-depth yearly interviews with teachers about their experiences in the project.

Development and Validation of the MKT Assessment

Following models of best practices in assessment development (e.g., AERA, APA, NCME, 1999), we took the following steps to develop and validate our instrument:

1. **Establish a conceptual framework.** We wanted to identify the specific mathematics knowledge that is important in teaching the unit. We went beyond “math necessary to teach the unit” and established a set of norms—what teachers ought to know to best teach the unit. These were established through an analysis of the mathematical domain and our curriculum materials, with central involvement of the developer of the replacement unit. Distilling this work, we identified the constructs that the instrument would assess:

    a. **MKT necessary for specific student learning goals.** For our student content assessment, we had established a map of the specific mathematical concepts students should learn during the unit. For each concept tested on the student assessment, the MKT instrument would assess (1) knowledge of the concept, and (2) specialized knowledge necessary to teach the concept. Specialized knowledge was separated into the following: knowledge underlying the student content, and knowledge that goes beyond the material taught in the unit (e.g. an understanding of how this material relates to further mathematics, such the relationship between position and velocity).

    b. **Connections.** These items would assess knowledge of conceptual connections within the unit and to other topics in mathematics and science.

2. **Generate a pool of items.** Many items exist to assess general content knowledge of rate and proportionality—more or less matching the student content, but with “harder” numbers and more complex problems. We had to generate our own pool of items to assess the knowledge necessary to teach rate and proportionality as outlined in our conceptual framework. We held a 1.5-day “item camp,” a workshop in which individuals with various types of expertise came together to collaboratively generate assessment items. The seven members of the item camp included the curriculum designer, a middle school math teacher, math education researchers, and assessment experts. During this workshop, about 80 items were generated.

3. **Assemble a pilot instrument.** The items generated in the item camp were subjected to review, refinement, and categorization with respect to our conceptual framework. We then constructed a 45-item pilot assessment with representation of each of the constructs in the framework.

4. **Field-test the pilot instrument.** We bought a mailing list of 1000 randomly selected middle school math teachers in public schools throughout the US. Teachers were sent two mailings. An initial mailing announced and described the MKT survey and the compensation of a $50 gift certificate. A week later, teachers received the 46-item assessment itself, along with an FAQ about the experiment, an informed waiver of consent, and a short demographic questionnaire. 179 teachers sent back the survey before the three-week deadline (18% response rate). We did not follow up with non-responders. On key demographic variables (gender, age, teaching experience, ethnicity, region type, and first language), the sample was representative of the population of teachers we expected to participate in the Scaling Up SimCalc Project, slightly oversampling suburban versus rural and urban regions.

5. **Cognitive think-alouds with the pilot assessment.** To evaluate the appropriateness of the responses that the items elicited (i.e., that the questions did, in fact, require and evoke proportional reasoning), we conducted cognitive think-alouds for each item in the assessment with three teachers. We used this data to refine or eliminate items that were off target with respect to the conceptual framework.

6. **Create final instrument**. One of the design requirements of the assessment was that it should take about 90 minutes in order to fit reasonably into the agenda of the teacher training workshops. The mean number of minutes spent on the pilot instrument was 88 (ranging from 36 to 180). This suggested that the test needed to be shorted considerably in order for the majority of teachers to complete it within the 90-minute time constraint.

   After dichotomously scoring each item, we examined IRT parameters for a 2-parameter logistic model (our sample size was insufficient for fitting an accurate 3-parameter model), item percent correct, analyses of errors, and think-aloud protocols. We used these sources of evidence to eliminate items that had low discrimination parameters (i.e., items that could not discriminate individuals of differing MKT ability), select (and occasionally modify) items that were likely to contribute the most information about a participant's MKT ability, and maintain representative coverage of the assessment conceptual framework. The resulting instrument has 24 items with an internal reliability of Cronbach's alpha=.86. Exploratory factor analysis shows one main factor accounting for 72% of the total variance.

Preliminary Findings

In the summer 2005 workshop, 117 teachers participated. Table 1 shows the basic demographic information about the sample. In these preliminary analyses, we discuss findings with respect to the first aims of the MKT assessment, to determine what relevant mathematics, if any, teachers learned during the summer workshops. As of April 2006, we are still collecting our student assessment data, so we cannot yet report findings with respect to the second aim of the MKT assessment, the extent to which teachers' MKT is related to student outcomes.

	Immediate Treatment	Delayed Treatment
Total count	58	59
Years teaching full time		
Mean	11.3	9.6
Minimum	0	1
Maximum	41	28
Age		
Mean	41.4	42.0
Minimum	24	26
Maximum	67	58
Gender		
% Female	77.4	84
% Male	22.6	16
Ethnicity		
% African American	2	0
% Asian	2	4
% Hispanic	24	22
% White	72	72
% Other	0	2

Table 1. Basic demographic information about the teacher sample.

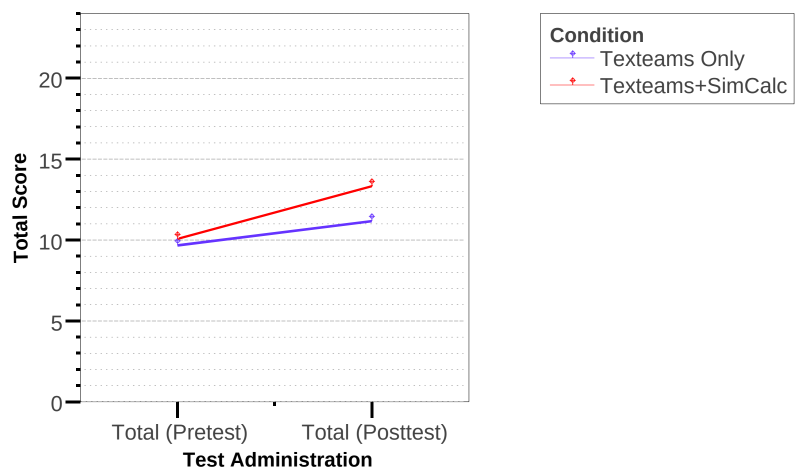


Figure 3. Teaching MKT scores pre- and post-workshop

Figure 3 shows teachers MKT scores pre- and post-workshop. Baseline scores were comparable in both groups [$F(1,115)=2.4$, $p=.128$, NS]. There was a significant main effect, such that both groups gained significantly [$F(1,115)=72.8$, $p<.001$]. The difference score was greater in the Texteams+SimCalc (Immediate Treatment) condition [$F(1,115)=10.2$, $p<.01$].

While significant, the average gain in the Immediate Treatment group was only about 3 points, smaller than we had expected. There are two possible explanations. One is that our instrument was not sensitive enough to detect learning; the other is that teachers were simply not afforded great opportunity to learn mathematics in their 5-day workshop. The latter explanation is the more plausible one, as only 1.5 days were focused on the mathematical content itself.

We will administer the assessment once again at the end of the second year (i.e., Spring 2007), after teachers have had the opportunity to teach with the unit and the software. We hypothesize that this experience will lead to an increase in their MKT.

Figure 4 shows that there is substantial variation in teacher MKT. This will be important as we explore relationships between teacher MKT and student outcomes.

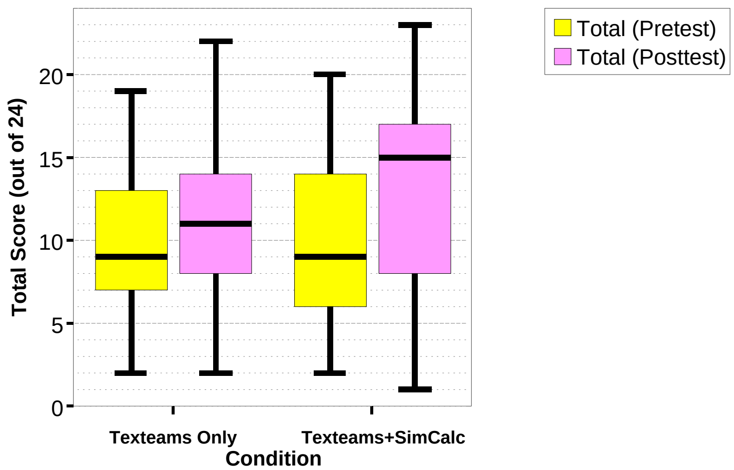


Figure 4. Boxplots illustrating the distributions of teachers' scores.

The Content Domain of the SimCalc Student Assessment

While the student assessment was designed to measure the knowledge, skills, and abilities outlined in this domain, the teacher MKT assessment was designed to measure the math knowledge necessary to *teach* these knowledge, skills, and abilities.

M_1 – Conceptually Simple Proportionality

Solving for a specific value

1.	Solving problems using the formula a/b=c/d	Simple a/b=c/d problem in which three of the values are provided and the fourth must be calculated or the proportion must be recognized
2.	Solving unit rate problems with y=kx or d=rt	Simple y=kx or d=rt problem in which two values are provided and a third must be calculated (even if it is based on own prior work)

Reading a specific value

3.	Basic graph reading of linear relationships	• Reading values at specific points, without interpreting their meaning as a rate • Using the labels of axes to determine the meaning of a given pair of x, y coordinates. • Sketching or plotting a given pair of x, y coordinates
4.	Basic table reading of linear relationships	Given a particular value, find the corresponding value in a table of a relationship.

M_2 – Complex and Conceptually Difficult

5.	Solving problems that invoke the function y=kx	Reasoning about a representation (e.g. graph, table or y=kx formula) in which a multiplicative constant "k" represents a constant rate, slope, speed, or scaling factor across many pairs of values (3 or more pairs) that are given or implied.

Within representations

6.	algebraic expression	Interpreting the behavior of a proportional function represented by an algebraic expression; or constructing an algebraic representation of a proportional function
7.	table	Filling in table cells of a table with many (3 or more pairs) values that are related by the same constant of proportionality
8.	graph	Interpreting or constructing the graph of a proportional or linear function.
9.	graph with a piecewise linear function	Interpreting or constructing a piecewise linear graph (e.g., with respect to narrative description of change over time)

Making connection(s) or comparison(s)

10.	across two or more functions	Interpreting, comparing, or constructing two or more linear or piecewise linear functions
11.	across multiple representations	Reasoning about the same proportional relationship across at least two of the following representations: graph, table, formula
12.	additive versus multiplicative	

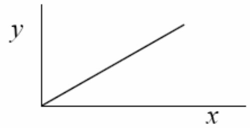
A Sample Item

18. The following are three representations of the same function:

A.

x	y
0	0
x_1	y_1
.	.
.	.
.	.
x_n	y_n

B.



C.

$y = kx$

Which of the statements below are **always true**? (*Mark [X]* ***ALL*** *that apply.*)

☐ A. $\frac{x_5}{y_5} = \frac{x_6}{y_6}$

☐ B. $\frac{x_5}{x_6} = \frac{y_5}{y_6}$

☐ C. $x_1 + x_2 = y_1 + y_2$

☐ D. $\frac{x_1}{y_1} = \mathrm{k}$

☐ E. The slope of the graph in B is equal to $\frac{y_1}{x_1}$

☐ F. $\frac{x_1}{x_2} = \mathrm{k}$

☐ G. You can find k with only one ordered pair from the line in graph B

This item was developed to assess whether teachers could link precise aspects of different representations of a proportional linear function, and do so using symbolic sophistication beyond what they are expected to use with their students. This was connected with our goal for students (as described in our student assessment content map): *Reasoning about the same proportional relationship across at least two of the following representations: graph, table, formula.* We expect that having this understanding of the connections will help guide students in understanding these connections at a similar level of precision (but stated in less symbolic language). This is particularly true when eliciting those connections from students, with teachers having to evaluate them “in the moment.” This item is an example of an item assessing knowledge not strictly necessary to teaching the unit with some success, but that does reflect relevant and reasonable expectations for teachers. In our field test this item did a good job of discriminating among individual teachers’ MKT, and so is a potentially important contributor in modeling the effect of teacher knowledge on student learning.

The item choices (we stress, in more symbolic terms than students would make them). represent connections students might make Because the function is a proportional relationship between two variables, students can correctly note that

A. You can make ratios with the x and y values in a row of the table. Take any two of those ratios, set them equal and you get a true proportion.
B. A true proportion can be created by comparing two different x values in the table to their corresponding y values.
C. It’s not true that any two x numbers added together will always equal the sum of their corresponding y values.
D. The ratio between any pair of numbers in the chart, when written x/y, is not the same as K in y=Kx (it is its multiplicative reciprocal though)
E. The slope of the line (k in y=kx) can be found by dividing any y by its corresponding x.
F. Dividing “consecutive” x values in the table doesn’t give you the constant of proportionality. (in fact it the quotient could be any number!)

References

American Educational Research Association, American Psychological Association, & National Council on Measurement in Education (AERA/APA/NCME) (2002). Standards for educational and psychological testing. Washington D.C.: American Educational Research Association.

Ball, D., & Hill, H. (n.d.). Developing measures of mathematics knowledge for teaching. Retrieved December 2, 2003 from http://soe.umich.edu/lmt/researchitems/index.html

Campbell, D. T., Shadish, W. R., & Cook, T. D. (2001). Experimental and quasi-experimental designs for generalized causal inference. New York: Houghton Mifflin Co.

Cohen, D. K., & Hill, H. (2001). Learning policy: When state education reform works. New Haven, CT: Yale University Press.

Greenes, C., & Findell, C. (1999). Developing students' algebraic reasoning abilities. In L. Stiff & F. R. Curcio (Eds.), Developing mathematical reasoning in grades K-12 (pp. 127-145). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Behr, M. (1988). Introduction: Capturing the major themes. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades (pp. 1-18). Hillsdale, NJ: Lawrence Erlbaum.

Kaput, J. (1994). Democratizing access to calculus: New routes using old roots. In A. Schoenfeld (Ed.), Mathematical thinking and problem solving (pp. 77-155). Hillsdale, NJ: Erlbaum.

Kaput, J., & Roschelle, J. (1998). The mathematics of change and variation from a millennial perspective: New content, new context. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), Rethinking the mathematics curriculum. London, UK: Falmer Press.

Kaput, J., & West, M. M. (1994). Missing-value proportional reasoning problems: Factors affecting informal reasoning patterns. In G. Harel & J. Confrey (Eds.), The development of multiplicative reasoning in the learning of mathematics. Albany, NY: State University of New York Press.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston, VA: Author.

Roschelle, J., & Kaput, J. (1996). SimCalc MathWorlds for the mathematics of change. Communications of the ACM, 39(8), 97-99.

Roschelle, J., Kaput, J., & Stroup, W. (2000). SimCalc: Accelerating students' engagement with the mathematics of change. In M. J. Jacobson & R. B. Kozma (Eds.), Innovation in science and math education: Advanced designs for technologies of learning. (pp. 47-75). Hillsdale, NJ: Erlbaum.

Slavin, R. E. (2002). Evidence-based education policies: Transforming educational practice and research. Educational Researcher, 31(7), 15-21.

Appendix: DRAFT Comparison of research-based studies investigating Math Knowledge for Teaching

Aspect of MKT	Scaling SimCalc	UMich (Ball-Hill)	MSU (Ferrini-Mundy)	Louisville (Hunt)
The mathematics content addressed	Middle school mathematics of change; particularly rate, slope, and proportionality	K-6 math focused on: Number concepts, operations and patterns; functions; and algebra.	Central algebraic concepts: themes that are foundational for understanding algebra, with a focus on algebraic expressions and equations; linear relationships	K-12 mathematics recognized as important by state and national standards, and education research
How the mathematics content was arrived at	Analyze the target curriculum unit and student assessment for the expected math knowledge required for teaching	Analyzing the mathematical knowledge needed to help students learn mathematics, using existing research, curriculum materials, examples of student work, and personal experience	Mathematics of Algebra	A review of standards and research
Additional knowledge addressed	Specialized math knowledge, including appraising non-standard solutions, use of representations and manipulatives, and explanations. Extensions of the core mathematics being taught	Specialized math knowledge, including appraising non-standard solutions, use of representations and manipulatives, and explanations	Specialized K for teaching, and tasks for teaching. These are summarized as Decompressing (expanding implicit K), trimming (keeping the important math), and bridging (various types of connecting and linking).	Facts and skills; conceptual knowledge; simple applied knowledge; relationships; structures; complex applied knowledge; and pedagogical content knowledge.
Target grade levels	Middle school	K-6	High School	K-12
Is pedagogy addressed	None	None	??	
Use	Norm referenced assessment; used in a model of student achievement	Norm referenced assessment: used to experimentally test impact of teacher knowledge	To create effective TPD materials (?)	Criterion referenced assessment: used to determine where teachers need professional development